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Reliability Estimation of Weibull Pareto Distribution Via Bayesian Approach

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Weibull Pareto distribution is considered. Bayesian method of estimation is employed in order to estimate the reliability function of Weibull Pareto distribution by using non-informative and beta priors. In this paper, the Bayes estimators of the reliability function have been obtained under squared error, precautionary and entropy loss functions.

Keywords: Weibull Pareto distribution, Reliability, Bayesian method, Non-informative and beta priors, Squared error, precautionary and entropy loss functions.

1. Introduction

The Weibull Pareto distribution has been proposed by Alzaatreh et al. [1]. They obtained the various properties of the distribution and proposed the method of modified maximum likelihood estimation for estimating the parameters. The probability function $f(x;\theta)$ and distribution function $F(x;\theta)$ of Weibull Pareto distribution are respectively given by

$$f(x;\theta) = b\theta x^{-1} \log\left(\frac{x}{a}\right)^{b-1} e^{-\theta\left[\log\left(\frac{x}{a}\right)\right]^{b}} \quad ; x > a > 0, b > 1. \tag{1}$$

$$F(x;\theta) = 1 - \exp\left\{-\theta\left[\log\left(\frac{x}{a}\right)\right]^{b}\right\} \quad ; x > a. \tag{2}$$

Let $R(t)$ denote the reliability function, that is, the probability that a system will survive a specified time t comes out to be

$$R(t) = exp\left\{-\theta\left[log\left(\frac{t}{a}\right)\right]^{b}\right\} \quad ; t > a. \tag{3}$$

And the instantaneous failure rate or hazard rate, h(t) is given by

$$h(t) = b\theta t^{-1}\left[log\left(\frac{t}{a}\right)\right]^{b-1}. \tag{4}$$

From equation (1) and (3), we get

$$f(x;R(t)) = \frac{b\left[log(x/a)\right]^{b-1}}{x\left[log(t/a)\right]^{b}}\left[-log\,R(t)\right]\left[R(t)\right]^{\left(\frac{log(x/a)}{log(t/a)}\right)^{b}} \quad ; \; 0<R(t)\leq 1. \tag{5}$$

The joint density function or likelihood function of (5) is given by

$$f(\underline{x}\,|\,R(t)) = \prod_{i=1}^{n}\left[\frac{b\left[log(x_i/a)\right]^{b-1}}{x_i\left[log(t/a)\right]^{b}}\right]\left[-log\,R(t)\right]^{n}\left[R(t)\right]^{\left(\sum_{i=1}^{n}\left[log(x_i/a)\right]^{b}/\left[log(t/a)\right]^{b}\right)} \tag{6}$$

The log likelihood function is given by

$$log\,f(\underline{x}\,|\,R(t)) = log\left(\prod_{i=1}^{n}\left[\frac{b\left[log(x_i/a)\right]^{b-1}}{x_i\left[log(t/a)\right]^{b}}\right]\right) + n\,log\left[-log\,R(t)\right] + \left(\sum_{i=1}^{n}\left[log(x_i/a)\right]^{b}/\left[log(t/a)\right]^{b}\right)log\left[R(t)\right] \tag{7}$$

Differentiating (7) with respect to R (t) and equating to zero, we get the maximum likelihood estimator of R (t) as

$$\hat{R}(t) = exp\left[-n\left\{\left[log(t/a)\right]^{b}\Big/\sum_{i=1}^{n}\left[log(x_i/a)\right]^{b}\right\}\right]. \tag{8}$$

2. Bayesian method of estimation

The Bayesian estimation procedure have been developed generally under squared error loss function

$$L\left(\hat{R}(t), R(t)\right) = \left(\hat{R}(t) - R(t)\right)^{2}. \tag{9}$$

where $\hat{R}(t)$ is an estimate of $R(t)$. The Bayes estimator under the above loss function, say $\hat{R}(t)_S$, is the posterior mean, i.e.,

$$\hat{R}(t)_S = E\left[R(t)\right]. \tag{10}$$

The squared error loss function is often used also because it does not lead extensive numerical computation but several authors (Zellner [2], Basu & Ebrahimi [3]) have recognized the inappropriateness of using symmetric loss function. Canfield [4] points out that the use of symmetric loss function may be inappropriate in the estimation of reliability function. Norstrom [5] introduced an alternative asymmetric precautionary loss function and also presented a general class of precautionary loss function with quadratic loss function as a special case. A very useful and simple asymmetric precautionary loss function is

$$L\left(\hat{R}(t), R(t)\right) = \frac{\left(\hat{R}(t) - R(t)\right)^2}{\hat{R}(t)} \tag{11}$$

The Bayes estimator of $R(t)$ under precautionary loss function is denoted by $\hat{R}(t)_P$, and is obtained by solving the following equation

$$\hat{R}(t)_P = \left[E\left(R(t)\right)^2\right]^{\frac{1}{2}}. \tag{12}$$

In many practical situations, it appears to be more realistic to express the loss in terms of the ratio $\frac{\hat{R}(t)}{R(t)}$. In this case, Calabria and Pulcini [6] points out that a useful asymmetric loss function is the entropy loss $L(\delta) \propto \left[\delta^p - p\, log_e(\delta) - 1\right]$, where $\delta = \hat{R}(t) \big/ R(t)$, and whose minimum occurs at $\hat{R}(t) = R(t)$ when $p > 0$, a positive error $\left(\hat{R}(t) > R(t)\right)$ causes more serious consequences than negative error, and vice-versa. For small $|p|$ value, the function is almost symmetric when both $\hat{R}(t)\, and\, R(t)$ are measured in a logarithmic scale, and approximately

$$L(\delta) \propto \frac{p^2}{2}\left[log_e \hat{R}(t) - log_e R(t)\right]^2.$$

Also, the loss function $L(\delta)$ has been used in Dey et al. [7] and Dey and Liu [8], in the original form having $p = 1$. Thus $L(\delta)$ can be written as

$$L(\delta) = b\left[\delta - log_e(\delta) - 1\right]; \quad b > 0. \tag{13}$$

The Bayes estimator of $R(t)$ under entropy loss function is denoted by $\hat{\theta}_E$ and is obtained as

$$\hat{R}(t)_E = \left[E\left(\frac{1}{R(t)}\right)\right]^{-1}. \tag{14}$$

For the situation where we have no prior information about $R(t)$, we may use non-informative prior distribution

$$h_1\left(R(t)\right) = \frac{1}{R(t)\; logR(t)}; \quad 0 < R(t) \le 1. \tag{15}$$

The most widely used prior distribution for $R(t)$ is a beta distribution with parameters $\alpha,\beta>0,$ given by

$$h_2\left(R(t)\right)=\frac{1}{B(\alpha,\beta)}\left[R(t)\right]^{\alpha-1}\left[1-R(t)\right]^{\beta-1};\quad 0<R(t)\le 1. \tag{16}$$

3. Bayes estimators of $R(t)$ under $h_1\left(R(t)\right)$

Under $h_1\left(R(t)\right)$, the posterior distribution is defined by

$$f\left(R(t)\mid \underline{x}\right)=\frac{f\left(\underline{x}\mid R(t)\right)h_1\left(R(t)\right)}{\int_0^1 f\left(\underline{x}\mid R(t)\right)h_1\left(R(t)\right)dR(t)} \tag{17}$$

Substituting the values of $h_1\left(R(t)\right)$ and $f\left(\underline{x}\mid R(t)\right)$ from equations (15) and (6) in (17), we get

$$f\left(R(t)\mid \underline{x}\right)=\frac{\left[\prod_{i=1}^{n}\left[\frac{b\left[log\left(x_i/a\right)\right]^{b-1}}{x_i\left[log\left(t/a\right)\right]^{b}}\right]\left[-log\,R(t)\right]^{n}\times\left[R(t)\right]^{\left(\sum_{i=1}^{n}\left[log(x_i/a)\right]^b/\left[log(t/a)\right]^b\right)}\frac{1}{R(t)\;log\,R(t)}\right]}{\int_0^1\left[\prod_{i=1}^{n}\left[\frac{b\left[log\left(x_i/a\right)\right]^{b-1}}{x_i\left[log\left(t/a\right)\right]^{b}}\right]\left[-log\,R(t)\right]^{n}\times\left[R(t)\right]^{\left(\sum_{i=1}^{n}\left[log(x_i/a)\right]^b/\left[log(t/a)\right]^b\right)}\frac{1}{R(t)\;log\,R(t)}\right]dR(t)}$$

$$=\frac{\left[R(t)\right]^{\left(\sum_{i=1}^{n}\left[log(x_i/a)\right]^b/\left[log(t/a)\right]^b\right)-1}\left[\text{-}log\,R(t)\right]^{n-1}}{\int_0^1\left[R(t)\right]^{\left(\sum_{i=1}^{n}\left[log(x_i/a)\right]^b/\left[log(t/a)\right]^b\right)-1}\left[\text{-}log\,R(t)\right]^{n-1}dR(t)}$$

or,
$$f\left(R(t)\mid \underline{x}\right)=\frac{\left(\sum_{i=1}^{n}\left[log\left(x_i/a\right)\right]^b/\left[log\left(t/a\right)\right]^b\right)^n}{\Gamma(n)}\left[R(t)\right]^{\left(\sum_{i=1}^{n}\left[log(x_i/a)\right]^b/\left[log(t/a)\right]^b\right)-1}\left[\text{-}log\,R(t)\right]^{n-1}$$

(18)

Theorem 1. Assuming the squared error loss function, the Bayes estimate of $R(t)$, is of the form

$$\hat{R}(t)_S=\left(1+\frac{\left[log\left(t/a\right)\right]^b}{\sum_{i=1}^{n}\left[log\left(x_i/a\right)\right]^b}\right)^{-n} \tag{19}$$

Proof. From equation (10), on using (18),

$$\hat{R}(t)_S = E\left[R(t)\right]$$

$$= \int_0^1 R(t) f\left(R(t) \mid \underline{x}\right) dR(t)$$

$$= \int_0^1 R(t) \frac{\left(\sum_{i=1}^{n}\left[log\left(x_i/a\right)\right]^b \Big/ \left[log\left(t/a\right)\right]^b\right)^n}{\Gamma(n)} \left[R(t)\right]^{\left(\sum_{i=1}^{n}\left[log\left(x_i/a\right)\right]^b / \left[log\left(t/a\right)\right]^b\right)-1} \left[-log\, R(t)\right]^{n-1} dR(t)$$

$$= \frac{\left(\sum_{i=1}^{n}\left[log\left(x_i/a\right)\right]^b \Big/ \left[log\left(t/a\right)\right]^b\right)^n}{\Gamma(n)} \int_0^1 \left[R(t)\right]^{\sum_{i=1}^{n}\left[log\left(x_i/a\right)\right]^b / \left[log\left(t/a\right)\right]^b} \left[-log\, R(t)\right]^{n-1} dR(t)$$

$$= \frac{\left(\sum_{i=1}^{n}\left[log\left(x_i/a\right)\right]^b \Big/ \left[log\left(t/a\right)\right]^b\right)^n}{\Gamma(n)} \frac{\Gamma(n)}{\left(\left(\sum_{i=1}^{n}\left[log\left(x_i/a\right)\right]^b \Big/ \left[log\left(t/a\right)\right]^b\right)+1\right)^n}$$

or, $$\hat{R}(t)_S = \left(1 + \frac{\left[log\left(t/a\right)\right]^b}{\sum_{i=1}^{n}\left[log\left(x_i/a\right)\right]^b}\right)^{-n} .$$

Theorem 2. Assuming the precautionary loss function, the Bayes estimate of $R(t)$, is of the form

$$\hat{R}(t)_P = \left[1 + \frac{2\left[log\left(t/a\right)\right]^b}{\sum_{i=1}^{n}\left[log\left(x_i/a\right)\right]^b}\right]^{-\frac{n}{2}} . \tag{20}$$

Proof. From equation (12), on using (18),

$$\hat{R}(t)_P = \left[E\left(R(t)\right)^2\right]^{\frac{1}{2}} = \left[\int_0^1 \left(R(t)\right)^2 f\left(R(t \mid \underline{x})\right) dR(t)\right]^{\frac{1}{2}}$$

$$= \left[\int_0^1 \left(R(t)\right)^2 \frac{\left(\sum_{i=1}^{n}\left[log\left(x_i/a\right)\right]^b \Big/ \left[log\left(t/a\right)\right]^b\right)^n}{\Gamma(n)} \left[R(t)\right]^{\left(\sum_{i=1}^{n}\left[log\left(x_i/a\right)\right]^b / \left[log\left(t/a\right)\right]^b\right)-1} \left[-log\, R(t)\right]^{n-1} dR(t)\right]^{\frac{1}{2}}$$

$$=\left[\frac{\left(\sum_{i=1}^{n}\left[log\left(x_i/a\right)\right]^b\Big/\left[log\left(t/a\right)\right]^b\right)^n}{\Gamma(n)}\int_0^1\left[R(t)\right]^{\left(\sum_{i=1}^{n}\left[log\left(x_i/a\right)\right]^b/\left[log\left(t/a\right)\right]^b\right)+1}\left[-log\,R(t)\right]^{n-1}dR(t)\right]^{\frac{1}{2}}$$

$$=\left[\frac{\left(\sum_{i=1}^{n}\left[log\left(x_i/a\right)\right]^b\Big/\left[log\left(t/a\right)\right]^b\right)^n}{\Gamma(n)}\;\frac{\Gamma(n)}{\left(\left(\sum_{i=1}^{n}\left[log\left(x_i/a\right)\right]^b\Big/\left[log\left(t/a\right)\right]^b\right)+2\right)^n}\right]^{\frac{1}{2}}$$

$$=\left[\frac{\left(\sum_{i=1}^{n}\left[log\left(x_i/a\right)\right]^b\Big/\left[log\left(t/a\right)\right]^b\right)^n}{\left(\left(\sum_{i=1}^{n}\left[log\left(x_i/a\right)\right]^b\Big/\left[log\left(t/a\right)\right]^b\right)+2\right)^n}\right]^{\frac{1}{2}}$$

or, $$\hat{R}(t)_P=\left[1+\frac{2\left[log\left(t/a\right)\right]^b}{\sum_{i=1}^{n}\left[log\left(x_i/a\right)\right]^b}\right]^{-\frac{n}{2}}.$$

Theorem 3. Assuming the entropy loss function, the Bayes estimate of $R(t)$, is of the form

$$\hat{R}(t)_E=\left[1-\frac{\left[log\left(t/a\right)\right]^b}{\sum_{i=1}^{n}\left[log\left(x_i/a\right)\right]^b}\right]^n \tag{21}$$

Proof. From equation (14), on using (18),

$$\hat{R}(t)_E=\left[E\left(\frac{1}{R(t)}\right)\right]^{-1}=\left[\int_0^1\frac{1}{R(t)}f\left(R(t\,|\,\underline{x})\right)dR(t)\right]^{-1}$$

$$=\left[\int_0^1\frac{1}{R(t)}\frac{\left(\sum_{i=1}^{n}\left[log\left(x_i/a\right)\right]^b\Big/\left[log\left(t/a\right)\right]^b\right)^n}{\Gamma(n)}\left[R(t)\right]^{\left(\sum_{i=1}^{n}\left[log\left(x_i/a\right)\right]^b/\left[log\left(t/a\right)\right]^b\right)-1}\left[-log\,R(t)\right]^{n-1}dR(t)\right]^{-1}$$

$$=\left[\frac{\left(\sum_{i=1}^{n}\left[log\left(x_i/a\right)\right]^b\Big/\left[log\left(t/a\right)\right]^b\right)^n}{\Gamma(n)}\int_0^1\left[R(t)\right]^{\left(\sum_{i=1}^{n}\left[log\left(x_i/a\right)\right]^b/\left[log\left(t/a\right)\right]^b\right)-2}\left[-log\,R(t)\right]^{n-1}dR(t)\right]^{-1}$$

$$=\left[\frac{\left(\sum_{i=1}^{n}\left[log\left(x_i/a\right)\right]^b\Big/\left[log\left(t/a\right)\right]^b\right)^n}{\Gamma(n)}\;\frac{\Gamma(n)}{\left(\left(\sum_{i=1}^{n}\left[log\left(x_i/a\right)\right]^b\Big/\left[log\left(t/a\right)\right]^b\right)-1\right)^n}\right]^{-1}$$

$$=\left[\frac{\left(\sum_{i=1}^{n}\left[log\left(x_i/a\right)\right]^b\Big/\left[log\left(t/a\right)\right]^b\right)^n}{\left(\left(\sum_{i=1}^{n}\left[log\left(x_i/a\right)\right]^b\Big/\left[log\left(t/a\right)\right]^b\right)-1\right)^n}\right]^{-1}$$

or, $$\hat{R}(t)_E=\left[1-\frac{\left[log\left(t/a\right)\right]^b}{\sum_{i=1}^{n}\left[log\left(x_i/a\right)\right]^b}\right]^n .$$

4. Bayes estimators of $R(t)$ under $h_2\left(R(t)\right)$

Under $h_2\left(R(t)\right)$, the posterior distribution is defined by

$$f\left(R(t)\,|\,\underline{x}\right)=\frac{f\left(\underline{x}\,|\,R(t)\right)h_2\left(R(t)\right)}{\int_0^1 f\left(\underline{x}\,|\,R(t)\right)h_2\left(R(t)\right)dR(t)} \tag{22}$$

Substituting the values of $h_2\left(R(t)\right)$ and $f\left(\underline{x}\,|\,R(t)\right)$ from equations (16) and (6) in (22), we get

$$f\left(R(t)\,|\,\underline{x}\right)=\frac{\left[\prod_{i=1}^{n}\left[\frac{b\left[log\left(x_i/a\right)\right]^{b-1}}{x_i\left[log\left(t/a\right)\right]^b}\right]\left[-log\,R(t)\right]^n\left[R(t)\right]^{\left(\sum_{i=1}^{n}\left[log(x_i/a)\right]^b/\left[log(t/a)\right]^b\right)}\times\frac{1}{B(\alpha,\beta)}\left[R(t)\right]^{\alpha-1}\left[1-R(t)\right]^{\beta-1}\right]}{\int_0^1\left[\prod_{i=1}^{n}\left[\frac{b\left[log\left(x_i/a\right)\right]^{b-1}}{x_i\left[log\left(t/a\right)\right]^b}\right]\left[-log\,R(t)\right]^n\left[R(t)\right]^{\left(\sum_{i=1}^{n}\left[log(x_i/a)\right]^b/\left[log(t/a)\right]^b\right)}\times\frac{1}{B(\alpha,\beta)}\left[R(t)\right]^{\alpha-1}\left[1-R(t)\right]^{\beta-1}\right]dR(t)}$$

$$=\frac{\left[R(t)\right]^{\left(\sum_{i=1}^{n}\left[log(x_i/a)\right]^b/\left[log(t/a)\right]^b\right)+\alpha-1}\left[\text{-}log\,R(t)\right]^n\left[1-R(t)\right]^{\beta-1}}{\int_0^1\left[R(t)\right]^{\left(\sum_{i=1}^{n}\left[log(x_i/a)\right]^b/\left[log(t/a)\right]^b\right)+\alpha-1}\left[\text{-}log\,R(t)\right]^n\left[1-R(t)\right]^{\beta-1}dR(t)}$$

or,

$$f\left(R(t)\,|\,\underline{x}\right)=\frac{\left[R(t)\right]^{\left(\sum_{i=1}^{n}\left[log(x_i/a)\right]^b/\left[log(t/a)\right]^b\right)+\alpha-1}\left[\text{-}log\,R(t)\right]^n\left[1-R(t)\right]^{\beta-1}}{\Gamma(n+1)\left[\sum_{k=0}^{\beta-1}(-1)^k\binom{\beta-1}{k}\left(1\Big/\left(\sum_{i=1}^{n}\left[log\left(x_i/a\right)\right]^b\Big/\left[log\left(t/a\right)\right]^b\right)+\alpha+k\right)^{n+1}\right]} \tag{23}$$

Theorem 4. Assuming the squared error loss function, the Bayes estimate of $R(t)$, is of the form

$$\hat{R}(t)_S = \left[\frac{\sum_{k=0}^{\beta-1}(-1)^k \binom{\beta-1}{k}\left(1\Big/\left(\sum_{i=1}^{n}\left[log(x_i/a)\right]^b\Big/\left[log(t/a)\right]^b\right)+\alpha+1+k\right)^{n+1}}{\sum_{k=0}^{\beta-1}(-1)^k \binom{\beta-1}{k}\left(1\Big/\left(\sum_{i=1}^{n}\left[log(x_i/a)\right]^b\Big/\left[log(t/a)\right]^b\right)+\alpha+k\right)^{n+1}}\right] \tag{24}$$

Proof. From equation (10), on using (23),

$$\hat{R}(t)_S = E\left[R(t)\right] = \int_0^1 R(t) f\left(R(t)\,|\,\underline{x}\right) dR(t)$$

$$= \int_0^1 R(t) \frac{\left[R(t)\right]^{\left(\sum_{i=1}^{n}\left[log(x_i/a)\right]^b/\left[log(t/a)\right]^b\right)+\alpha-1}\left[\text{-}log\,R(t)\right]^n\left[1-R(t)\right]^{\beta-1}}{\Gamma(n+1)\left[\sum_{k=0}^{\beta-1}(-1)^k \binom{\beta-1}{k}\left(1\Big/\left(\sum_{i=1}^{n}\left[log(x_i/a)\right]^b\Big/\left[log(t/a)\right]^b\right)+\alpha+k\right)^{n+1}\right]} dR(t)$$

$$= \frac{\int_0^1 \left[R(t)\right]^{\left(\sum_{i=1}^{n}\left[log(x_i/a)\right]^b/\left[log(t/a)\right]^b\right)+\alpha}\left[\text{-}log\,R(t)\right]^n\left[1-R(t)\right]^{\beta-1} dR(t)}{\Gamma(n+1)\left[\sum_{k=0}^{\beta-1}(-1)^k \binom{\beta-1}{k}\left(1\Big/\left(\sum_{i=1}^{n}\left[log(x_i/a)\right]^b\Big/\left[log(t/a)\right]^b\right)+\alpha+k\right)^{n+1}\right]}$$

or, $$\hat{R}(t)_S = \left[\frac{\sum_{k=0}^{\beta-1}(-1)^k \binom{\beta-1}{k}\left(1\Big/\left(\sum_{i=1}^{n}\left[log(x_i/a)\right]^b\Big/\left[log(t/a)\right]^b\right)+\alpha+1+k\right)^{n+1}}{\sum_{k=0}^{\beta-1}(-1)^k \binom{\beta-1}{k}\left(1\Big/\left(\sum_{i=1}^{n}\left[log(x_i/a)\right]^b\Big/\left[log(t/a)\right]^b\right)+\alpha+k\right)^{n+1}}\right].$$

Theorem 5. Assuming the precautionary loss function, the Bayes estimate of $R(t)$, is of the form

$$\hat{R}(t)_P = \left[\frac{\sum_{k=0}^{\beta-1}(-1)^k \binom{\beta-1}{k}\left(1\Big/\left(\sum_{i=1}^{n}\left[log(x_i/a)\right]^b\Big/\left[log(t/a)\right]^b\right)+\alpha+2+k\right)^{n+1}}{\sum_{k=0}^{\beta-1}(-1)^k \binom{\beta-1}{k}\left(1\Big/\left(\sum_{i=1}^{n}\left[log(x_i/a)\right]^b\Big/\left[log(t/a)\right]^b\right)+\alpha+k\right)^{n+1}}\right]^{\frac{1}{2}} \tag{25}$$

Proof. From equation (12), on using (23),

$$\hat{R}(t)_P = \left[E\left(R(t)\right)^2\right]^{\frac{1}{2}}$$

$$= \left[\int_0^1 \left(R(t)\right)^2 f\left(R(t\,|\,\underline{x})\right) dR(t)\right]^{\frac{1}{2}}$$

$$= \left[\int_0^1 \left(R(t)\right)^2 \frac{\left[R(t)\right]^{\left(\sum_{i=1}^{n}\left[log(x_i/a)\right]^b / \left[log(t/a)\right]^b\right)+\alpha-1} \left[\text{-}log\, R(t)\right]^n \left[1-R(t)\right]^{\beta-1}}{\Gamma(n+1)\left[\sum_{k=0}^{\beta-1}(-1)^k \binom{\beta-1}{k}\left(1\Big/\left(\sum_{i=1}^{n}\left[log(x_i/a)\right]^b \Big/ \left[log(t/a)\right]^b\right)+\alpha+k\right)^{n+1}\right]} dR(t) \right]^{\frac{1}{2}}$$

$$= \left[\frac{\int_0^1 \left[R(t)\right]^{\left(\sum_{i=1}^{n}\left[log(x_i/a)\right]^b / \left[log(t/a)\right]^b\right)+\alpha+1} \left[\text{-}log\, R(t)\right]^n \left[1-R(t)\right]^{\beta-1} dR(t)}{\Gamma(n+1)\left[\sum_{k=0}^{\beta-1}(-1)^k \binom{\beta-1}{k}\left(1\Big/\left(\sum_{i=1}^{n}\left[log(x_i/a)\right]^b \Big/ \left[log(t/a)\right]^b\right)+\alpha+k\right)^{n+1}\right]} \right]^{\frac{1}{2}}$$

or, $$\hat{R}(t)_P = \left[\frac{\sum_{k=0}^{\beta-1}(-1)^k \binom{\beta-1}{k}\left(1\Big/\left(\sum_{i=1}^{n}\left[log(x_i/a)\right]^b \Big/ \left[log(t/a)\right]^b\right)+\alpha+2+k\right)^{n+1}}{\sum_{k=0}^{\beta-1}(-1)^k \binom{\beta-1}{k}\left(1\Big/\left(\sum_{i=1}^{n}\left[log(x_i/a)\right]^b \Big/ \left[log(t/a)\right]^b\right)+\alpha+k\right)^{n+1}} \right]^{\frac{1}{2}}.$$

Theorem 6. Assuming the entropy loss function, the Bayes estimate of $R(t)$, is of the form

$$\hat{R}(t)_E = \left[\frac{\sum_{k=0}^{\beta-1}(-1)^k \binom{\beta-1}{k}\left(1\Big/\left(\sum_{i=1}^{n}\left[log(x_i/a)\right]^b \Big/ \left[log(t/a)\right]^b\right)+\alpha+k\right)^{n+1}}{\sum_{k=0}^{\beta-1}(-1)^k \binom{\beta-1}{k}\left(1\Big/\left(\sum_{i=1}^{n}\left[log(x_i/a)\right]^b \Big/ \left[log(t/a)\right]^b\right)+\alpha-1+k\right)^{n+1}} \right] \tag{26}$$

Proof. From equation (14), on using (23),

$$\hat{R}(t)_E = \left[E\left(\frac{1}{R(t)}\right)\right]^{-1} = \left[\int_0^1 \frac{1}{R(t)} f\left(R(t\,|\,\underline{x})\right) dR(t)\right]^{-1}$$

$$= \left[\int_0^1 \frac{1}{R(t)} \frac{\left[R(t)\right]^{\left(\sum_{i=1}^{n}\left[log(x_i/a)\right]^b / \left[log(t/a)\right]^b\right)+\alpha-1} \left[\text{-}log\, R(t)\right]^n \left[1-R(t)\right]^{\beta-1}}{\Gamma(n+1)\left[\sum_{k=0}^{\beta-1}(-1)^k \binom{\beta-1}{k}\left(1\Big/\left(\sum_{i=1}^{n}\left[log(x_i/a)\right]^b \Big/ \left[log(t/a)\right]^b\right)+\alpha+k\right)^{n+1}\right]} dR(t) \right]^{-1}$$

$$= \left[\frac{\int_0^1 \left[R(t)\right]^{\left(\sum_{i=1}^{n}\left[log(x_i/a)\right]^b / \left[log(t/a)\right]^b\right)+\alpha-2} \left[\text{-}log\, R(t)\right]^n \left[1-R(t)\right]^{\beta-1} dR(t)}{\Gamma(n+1)\left[\sum_{k=0}^{\beta-1}(-1)^k \binom{\beta-1}{k}\left(1\Big/\left(\sum_{i=1}^{n}\left[log(x_i/a)\right]^b \Big/ \left[log(t/a)\right]^b\right)+\alpha+k\right)^{n+1}\right]} \right]^{-1}$$

$$= \left[\frac{\sum_{k=0}^{\beta-1}(-1)^k \binom{\beta-1}{k}\left(1\Big/\left(\sum_{i=1}^{n}\left[log(x_i/a)\right]^b \Big/ \left[log(t/a)\right]^b\right)+\alpha-1+k\right)^{n+1}}{\sum_{k=0}^{\beta-1}(-1)^k \binom{\beta-1}{k}\left(1\Big/\left(\sum_{i=1}^{n}\left[log(x_i/a)\right]^b \Big/ \left[log(t/a)\right]^b\right)+\alpha+k\right)^{n+1}} \right]^{-1}$$

or, $$\hat{R}(t)_E = \left[\frac{\sum_{k=0}^{\beta-1}(-1)^k\binom{\beta-1}{k}\left(1\Big/\left(\sum_{i=1}^{n}\left[log\left(x_i/a\right)\right]^b\Big/\left[log\left(t/a\right)\right]^b\right)+\alpha+k\right)^{n+1}}{\sum_{k=0}^{\beta-1}(-1)^k\binom{\beta-1}{k}\left(1\Big/\left(\sum_{i=1}^{n}\left[log\left(x_i/a\right)\right]^b\Big/\left[log\left(t/a\right)\right]^b\right)+\alpha-1+k\right)^{n+1}}\right].$$

5. Conclusion

We have obtained a number of Bayes estimators of reliability function $R(t)$ of Weibull Pareto distribution. In equations (19), (20), and (21), we have obtained the Bayes estimators by using non-informative prior and in equations (24), (25), and (26), under beta prior. From the above said equation, it is clear that the Bayes estimators of $R(t)$ depend upon the parameters of the prior distribution.

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